

Main phenomenological features of the spontaneous CP violation in $SU(3)_L \otimes U(1)_Y$ models

D. Gómez Dumm*

Laboratorio de Física Teórica,
Departamento de Física, U.N.L.P.,
c.c. 67, 1900, La Plata, Argentina[†]

Abstract

We analyze the phenomenological consequences of assuming spontaneous CP violation in an $SU(3)_L \otimes U(1)_Y$ model with three Higgs triplets and one sextuplet. After the identification of the relevant physical scalars, we estimate the contributions to the parameters Δm_K , ε and ε' coming from the Higgs-fermion couplings.

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*Fellow of CONICET (Argentina)

[†]e-mail: dumm@venus.fisica.unlp.edu.ar

Introduction

Since the discovery of CP violation in 1964, the understanding of its origin has represented a very attractive problem from the theoretical point of view. Within the context of the Standard Model (SM), the present experimental situation can be successfully explained through the well-known Kobayashi–Maskawa mechanism. This requires explicit CP non-conservation at the Lagrangian level. However, as it was firstly pointed out by T. D. Lee [1], the CP symmetry can also be spontaneously broken if the SM Higgs sector is extended.

In this work, we consider an extension of the SM based on the $SU(3)_C \otimes SU(3)_L \otimes U(1)_Y$ gauge symmetry group, which has been recently proposed by Pisano and Pleitez [2] and by Frampton [3]. The fermions are here organized into $SU(3)_L$ triplets and antitriplets, in such a way that the model is anomaly-free only when the number of lepton families is a multiple of the number of colors. The $SU(3)_L$ structure leads to the introduction of new unobserved particles, such as exotic “leptoquarks” and new vector bosons of double, single and zero electric charge. In addition, a large number of scalar fields is required in order to provide the fermion masses.

In a previous paper [4], we studied the possibility of finding spontaneous CP violation within this scenario^a. By analyzing the Higgs potential, we have shown that the mechanism can be triggered for a certain range of the involved parameters. This could give rise to measurable effects through the presence of complex couplings in the Yukawa Lagrangian.

The purpose of this article is to analyze the phenomenology originated by the nontrivial structure of the neutral and single charged Higgs sector of the model. We study the effects of scalar-mediated flavor-changing neutral interactions (FCNI’s), as well as the possibility of spontaneous CP violation. In particular, we show that this scheme is capable of explaining the observed CP violation phenomena related to kaon decays.

^aThe problem of CP violation in this kind of model has been also treated in Ref. [5], in connection with lepton-number-violating interactions.

We proceed first by identifying the scalar mass eigenstates when CP is conserved and then when CP is spontaneously broken. Afterwards we compute the contributions to the mass difference $m_{K_L} - m_{K_S}$ and the CP-violating parameters ε and ε' . Finally, we discuss the compatibility of the model with the experimental results.

Fermions, scalars and gauge bosons

As was mentioned above, the left-handed fermions appear organized into $SU(3)_L$ triplets. These are

$$\Psi_{lL} = \begin{pmatrix} \nu_l \\ l \\ l^c \end{pmatrix}_L \sim (\underline{3}, 0) \quad Q_{1L} = \begin{pmatrix} u_1 \\ d_1 \\ J_1 \end{pmatrix}_L \sim (\underline{3}, \frac{2}{3})$$

$$Q_{2L} = \begin{pmatrix} J_2 \\ u_2 \\ d_2 \end{pmatrix}_L, \quad Q_{3L} = \begin{pmatrix} J_3 \\ u_3 \\ d_3 \end{pmatrix}_L \sim (\underline{3}^*, -\frac{1}{3})$$

where $l = e, \mu, \tau$. The first and second entries following each triplet denote the corresponding $SU(3)_L$ representation and Y quantum number respectively. Here, in order to complete the quark triplets, the inclusion of “leptoquarks” of charge $\frac{5}{3}$ and $-\frac{4}{3}$ has been necessary. Notice that two of the quark families belong to the $\underline{3}$ representation, while the third one transforms as a $\underline{3}^*$. This gives rise to FCNI’s at the tree level.

The right-handed fields are introduced in the model as $SU(3)_L$ singlets

$$u_{iR} \sim (\underline{1}, \frac{2}{3}) \quad d_{iR} \sim (\underline{1}, -\frac{1}{3}) \quad J_{1R} \sim (\underline{1}, \frac{5}{3}) \quad J_{2,3R} \sim (\underline{1}, -\frac{4}{3}) \quad l_R \sim (\underline{1}, -1)$$

In both cases, the values of Y verify the Gell-Mann–Nishijima relation

$$Q = Y + T_3 - \sqrt{3}T_8$$

The fermions acquire masses through Yukawa-like couplings with Higgs bosons. These scalar fields are arranged into three $SU(3)_L$ triplets with Y values 1, 0 and -1 respectively,

$$\rho = \begin{pmatrix} \rho^+ \\ \rho^0 \\ \rho^{++} \end{pmatrix}, \quad \eta = \begin{pmatrix} \eta^0 \\ \eta_1^- \\ \eta_2^+ \end{pmatrix}, \quad \chi = \begin{pmatrix} \chi^- \\ \chi^{--} \\ \chi^0 \end{pmatrix}$$

and one $Y=0$ sextuplet

$$S = \begin{pmatrix} \sigma_1^0 & h_2^- & h_1^+ \\ h_2^- & H_1^{--} & \sigma_2^0 \\ h_1^+ & \sigma_2^0 & H_2^{++} \end{pmatrix}$$

Finally, the model also contains new charged vector bosons, namely V^+ and U^{++} , which together with their charge conjugated and a new neutral vector field Z'^0 complete the $SU(3)_L$ gauge octet.

Higgs potential and CP violation

In order to yield the expected spontaneous gauge symmetry breakdown, the scalar potential should be minimized when the scalar fields take the values

$$\langle \rho \rangle = \begin{pmatrix} 0 \\ v_\rho e^{i\theta_\rho} \\ 0 \end{pmatrix}, \quad \langle \eta \rangle = \begin{pmatrix} v_\eta e^{i\theta_\eta} \\ 0 \\ 0 \end{pmatrix}, \quad \langle \chi \rangle = \begin{pmatrix} 0 \\ 0 \\ v_\chi e^{i\theta_\chi} \end{pmatrix}, \quad \langle S \rangle = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & v_s e^{i\theta_s} \\ 0 & v_s e^{i\theta_s} & 0 \end{pmatrix} \quad (1)$$

where we have explicitly written the nonzero VEV's as complex numbers. If the gauge symmetry of \mathcal{L} is now taken into account, the phases θ_χ and θ_ρ can be chosen to be equal to zero without any loss of generality. The remaining phases θ_η and θ_s survive as CP-violating parameters in the final Lagrangian of the theory.

In (1) we have chosen $\langle \sigma_1^0 \rangle = 0$, preventing in this way the presence of neutrino mass terms. It can be shown, however, that if $\langle \sigma_1^0 \rangle$ has to be kept equal to zero, a fine tuning between some of the coupling constants in the Higgs potential $V(\eta, \rho, \chi, S)$ [4,6] becomes necessary. One way of avoiding this fine tuning and achieving the above vacuum expectation values is through the introduction of a particular set of discrete symmetries [6]:

$$\begin{aligned}
Q_{1L} &\rightarrow -Q_{1L} & \eta &\rightarrow -\eta \\
Q_{jL} &\rightarrow -iQ_{jL} & \rho, \chi &\rightarrow i \rho, \chi \\
\Psi_{lL} &\rightarrow i\Psi_{lL} & S &\rightarrow -S \\
u_{jR} &\rightarrow u_{jR} & J_{1R} &\rightarrow iJ_{1R} \\
d_{jR} &\rightarrow id_{jR} & J_{2,3R} &\rightarrow J_{2,3R}
\end{aligned} \tag{2}$$

where $j = 1, 2, 3$ and $l = e, \mu, \tau$. At the same time, the presence of these symmetries ensures that the lepton and baryon numbers result conserved in the final Lagrangian separately.

Once we have obtained the desired nonzero VEV's in (1), we demand them to satisfy the relations

$$v_\chi^2 \gg v_\eta^2 \approx v_\rho^2 \approx v_s^2 \tag{3}$$

in order to guarantee that the gauge symmetry breakdown follows the hierarchy $SU(3)_L \otimes U(1)_Y \rightarrow SU(2)_L \otimes U(1)_Y \rightarrow U(1)_{em}$. In addition, from the Higgs kinetic term in \mathcal{L} these VEV's are related by

$$\frac{g}{2} \left(v_\eta^2 + v_\rho^2 + \frac{v_s^2}{2} \right) = m_W^2 \tag{4}$$

Let us now turn to the phenomenology of the model, in particular to those effects coming from the enlarged scalar sector. To this end, we first need to isolate the physical scalars of the theory, which result from the diagonalization of the Higgs mass matrices. Notice that the latter receive contributions from terms which are proportional to $(\varphi_i^\dagger \varphi_i)$, $(\varphi_i^\dagger \varphi_i)(\varphi_j^\dagger \varphi_j)$, and others involving the product $S^\dagger S$, φ_i being any of the Higgs triplets and S the Higgs sextuplet. Considering the symmetries in (2), the remaining contributions come from the non-self-Hermitian terms present in the potential,

$$\begin{aligned}
&f_1 \mu_1 \epsilon_{ijk} \eta_i \rho_j \chi_k + f_2 \mu_2 \chi^\dagger S \rho^* + f_3 \epsilon_{ijk} \eta_i \rho_j (S \rho^*)_k \\
&+ f_4 \epsilon_{ijk} \eta_i (S \chi^*)_j \chi_k + f_5 \epsilon_{ijk} \epsilon_{lmn} S_{il} S_{jm} \eta_k \eta_n + \text{h.c.}
\end{aligned} \tag{5}$$

where the parameters $\mu_{1,2}$ have dimensions of mass. As is shown in Ref. [4], this non-self-Hermitian piece of \mathcal{L} yields spontaneous CP violation for an appropriate range of the parameters f_i and μ_i .

The main contributions to Δm_K and ε are expected to come from the neutral flavor changing interactions, therefore we start with the analysis of the neutral scalar mass matrix. Due to the symmetries (2), the fields $\text{Re}(\sigma_1^0)$ and $\text{Im}(\sigma_1^0)$ decouple to the rest, becoming exact mass eigenstates. The other neutral scalar fields, namely the real and imaginary parts of η^0 , ρ^0 , χ^0 and σ_2^0 , result mixed by an 8×8 real mass matrix. As a first step, we identify in this sector two $m = 0$ eigenstates, the neutral scalar Goldstone bosons. These are given by

$$\begin{aligned} G_1^0 &= \frac{1}{\beta} \left[v_\rho \text{Im}(\rho^0) - v_\eta \text{Im}(\eta^0 e^{-i\theta_\eta}) + v_s \text{Im}(\sigma_2^0 e^{-i\theta_s}) \right] \\ G_2^0 &= \frac{1}{\beta N^2} \left[-v_\rho \alpha^2 \text{Im}(\rho^0) - v_\rho^2 v_\eta \text{Im}(\eta^0 e^{-i\theta_\eta}) + v_\rho^2 v_s \text{Im}(\sigma_2^0 e^{-i\theta_s}) + \beta^2 v_x \text{Im}(\chi^0) \right] \end{aligned} \quad (6)$$

where we have defined $\alpha = (v_\eta^2 + v_s^2)^{\frac{1}{2}}$, $\beta = (\alpha^2 + v_\rho^2)^{\frac{1}{2}}$, $N^2 = (\alpha^2 v_\rho^2 + \beta^2 v_\chi^2)^{\frac{1}{2}}$.

The identification of the massive scalar fields is now necessary. This is not a trivial task, in view of the many terms which are present in the Higgs potential. However, it is possible to get more insight into the physics of this sector if one considers some proper assumptions.

Let us present the potential in such a way that the terms of the form $(\varphi_i^\dagger \varphi_i)(\varphi_j^\dagger \varphi_j)$, appear written as $\lambda_{ij}(\varphi_i^\dagger \varphi_i - |v_i|^2 + \varphi_j^\dagger \varphi_j - |v_j|^2)^2$, and analogously for the terms involving $S^\dagger S$ [6]. The basic assumption will be to consider that no further scales are introduced by the parameters in the Higgs potential once it has been written in this “complete squared” way. That is, we take all the λ_{ij} parameters, together with the f_i ones, to be of the same order of magnitude, say $\mathcal{O}(\lambda)$. Then, taking into account the relation (3), we will suppose that the “massive” parameters $\mu_{1,2}$ lay in the range $v_\eta < \mu_i < v_\chi$, so that no other mass scale is needed.

Next, we change to the new basis

$$\phi_1 = \frac{1}{\beta} \left[v_\eta \text{Re}(\eta^0 e^{-i\theta_\eta}) + v_\rho \text{Re}(\rho^0) + v_s \text{Re}(\sigma_2^0 e^{-i\theta_s}) \right]$$

$$\begin{aligned}
\phi_2 &= \text{Re}(\chi^0) \\
\phi_3 &= \frac{1}{\beta\alpha} \left[-v_\eta v_\rho \text{Re}(\eta^0 e^{-i\theta_\eta}) + \alpha^2 \text{Re}(\rho^0) - v_s v_\rho \text{Re}(\sigma_2^0 e^{-i\theta_s}) \right] \\
\phi_4 &= \frac{1}{\alpha N^2} \left[v_\eta v_\rho v_\chi \text{Im}(\eta^0 e^{-i\theta_\eta}) + v_\chi \alpha^2 \text{Im}(\rho^0) - v_s v_\rho v_\chi \text{Im}(\sigma_2^0 e^{-i\theta_s}) + v_\rho \alpha^2 \text{Im}(\chi^0) \right] \\
\phi_5 &= \frac{1}{\alpha} \left[-v_s \text{Re}(\eta^0 e^{-i\theta_\eta}) + v_\eta \text{Re}(\sigma_2^0 e^{-i\theta_s}) \right] \\
\phi_6 &= \frac{1}{\alpha} \left[v_s \text{Im}(\eta^0 e^{-i\theta_\eta}) + v_\eta \text{Im}(\sigma_2^0 e^{-i\theta_s}) \right]
\end{aligned} \tag{7}$$

It can be seen that these will be the approximate mass eigenstates when $v_\eta \ll \mu \ll v_\chi$. The respective masses will follow in this case the hierarchy

$$\begin{aligned}
m_{\phi_1}^2 &\sim \lambda v_\eta^2 \\
m_{\phi_4}^2, m_{\phi_3}^2 &\sim \lambda \mu v_\chi \\
m_{\phi_5}^2, m_{\phi_6}^2, m_{\phi_2}^2 &\sim \lambda v_\chi^2
\end{aligned} \tag{8}$$

Setting $\theta_\eta = \theta_s = 0$, we see that ϕ_4 and ϕ_6 have to be CP-odd, while the other scalar fields are CP even.

Let us now examine the quark-Higgs Yukawa couplings of the Lagrangian. In terms of the $SU(3)_L$ multiplets components, we have for the neutral Higgs sector

$$\begin{aligned}
-\mathcal{L}_Y &= \frac{1}{v_\rho^*} \rho^{0*} \bar{U}_L \hat{M}^u U_R + \frac{1}{v_\eta} \left(\eta^0 - \frac{v_\eta}{v_\rho^*} \rho^{0*} \right) \bar{U}_L \Delta^u \hat{M}^u U_R \\
&\quad + \frac{1}{v_\eta^*} \eta^{0*} \bar{D}_L \hat{M}^d D_R + \frac{1}{v_\rho} \left(\rho^0 - \frac{v_\rho}{v_\eta^*} \eta^{0*} \right) \bar{D}_L \Delta^d \hat{M}^d D_R + \text{h.c.}
\end{aligned} \tag{9}$$

where \hat{M}^u and \hat{M}^d represent the diagonalized quark mass matrices, and we have also defined $\Delta^{u,d} \equiv V_L^{u,d\dagger} \text{diag}(1, 0, 0) V_L^{u,d}$. The unitary matrices $V_L^{u,d}$, used for changing to the quark mass eigenbasis, are related by

$$V_L^{u\dagger} V_L^d = V_{CKM} \tag{10}$$

From (9), the neutral Higgs sector is nondiagonal in flavor. We expect then to find significant contributions to Δm_K and ε coming from tree level FCNI's.

If we choose now the basis defined in (7) for the scalar fields, the Yukawa couplings take the form

$$\begin{aligned}\mathcal{L}_Y = & \frac{1}{\beta}(\bar{U}\hat{M}^u U)\phi_1 + \left[\frac{\alpha}{\beta v_\rho}\bar{U}\hat{M}^u U - \frac{\beta}{\alpha v_\rho}\bar{U}(A_+^u + A_-^u\gamma_5)U \right] \phi_3 \\ & + i \left[-\frac{\alpha}{\beta v_\rho}\bar{U}\hat{M}^u\gamma_5 U + \frac{\beta}{\alpha v_\rho}\bar{U}(A_-^u + A_+^u\gamma_5)U \right] \phi_4 + (u \leftrightarrow d)\end{aligned}\quad (11)$$

where $A_\pm^u = \Delta^u \hat{M}^u \pm \hat{M}^u \Delta^u$. From this expression, the identification of the ϕ_1 as the “standard” flavor conserving Higgs is immediate. The lowest (tree level) correction to Δm_K is given by the scalars ϕ_4 and ϕ_3 , which have intermediate masses (see Eq. (8)). On the other hand, according to the CP parity for the involved neutral fields, \mathcal{L}_Y is found to be CP invariant, not depending on θ_η and θ_s . In this way, the CP violation effects will be due to the mixing among the scalars, hence they will be suppressed at least by a mass scale ratio such as μ/v_χ or v_η/μ .

We will look into the following for the possibility of finding spontaneous CP breakdown. We have shown in Ref. [4] that this can be achieved only if the mass parameters μ_i are close to the VEV v_χ . This enables the presence of just two relevant mass scales in the Higgs sector, say v_χ and v_η . The Higgs potential will be minimized in this case by nonzero values of θ_η and θ_s . These values can be exactly calculated in terms of v_η , v_ρ , v_χ , v_s and the potential parameters f_i and μ_i [4].

Once the relation $\mu \sim v_\chi$ is assumed, the fields defined in (7) are no longer approximate mass eigenstates. In order to find the exact physical particles, let us once again change to a new basis, namely

$$\begin{aligned}h_2^{0'} &= \frac{1}{N_2^2} [-c\phi_3 + b\phi_4 + d\phi_6] \\ h_3^{0'} &= \frac{1}{N_4^2} [-d\phi_3 - c\frac{\beta v_\chi}{N^2}\phi_5 + b\phi_6] \\ H_2^{0'} &= \frac{1}{N_1^2} [a\phi_3 + b\phi_4 + d\phi_6] \\ H_3^{0'} &= \frac{1}{N_3^2} [-d\phi_3 + a\frac{\beta v_\chi}{N^2}\phi_5 + b\phi_6] \\ h_1^{0'} &= \phi_1 \\ H_1^{0'} &= \phi_2\end{aligned}\quad (12)$$

where the N_i^{-2} are normalization factors, verifying $N_1 \simeq N_3$ and $N_2 \simeq N_4$. The parameters a , b , c and d are functions of the coupling constants f_i and μ_i :

$$\begin{aligned}
a &= \frac{2\beta^2 v_\chi}{v_\rho \alpha^2} (f_1 \mu_1 v_\eta \cos \theta_\eta + f_2 \mu_2 v_s \cos \theta_s) \\
b &= \frac{2\beta v_\chi}{\alpha^2} (f_1 \mu_1 v_s \cos \theta_\eta - f_2 \mu_2 v_\eta \cos \theta_s) \\
c &= f_4 \frac{2\alpha^2 v_\chi^2}{v_\eta v_s} \cos(\theta_\eta + \theta_s) + \frac{2v_\chi v_\rho}{\alpha^2} (f_1 \mu_1 \frac{v_s^2}{v_\eta} \cos \theta_\eta + f_2 \mu_2 \frac{v_\eta^2}{v_s} \cos \theta_s) \\
d &= f_1 \mu_1 \frac{2v_\chi \beta}{v_s} \sin \theta_\eta
\end{aligned} \tag{13}$$

The CP violation is carried by the $\sin \theta_\eta$ in d .

It is now worth regarding the appearance of the mass matrix. Actually, it is found that it blocks out into two parts, which correspond to three $m^2 \sim \lambda v_\eta^2$ fields and three $m^2 \sim \lambda v_\chi^2$ fields, combinations of the $h_i^{0'}$ and the $H_i^{0'}$ ones respectively. If we ignore the (v_η^2/v_χ^2) corrections due to the $h_i^{0'} - H_i^{0'}$ mixing, we obtain finally the physical particles

$$h_i^0 = \mathcal{U}_{ij} h_j^{0'} \quad H_i^0 = \mathcal{V}_{ij} H_j^{0'} \tag{14}$$

whose masses remain of order $\lambda^{\frac{1}{2}} v_\eta$ and $\lambda^{\frac{1}{2}} v_\chi$ respectively. When the Lagrangian (11) is written in terms of the approximate mass eigenstates h_i^0 and H_i^0 defined above, one can immediately see that all the resulting neutral couplings are in general nondiagonal in flavor.

We focus our attention on the single charged scalars now. Due to the symmetries (2), these separate into two groups, the η_1^+, ρ^+, h_1^+ and the η_2^+, χ^+, h_2^+ fields. The second group does not interact with the ordinary quarks at the tree level and will not be considered here.

Following a similar procedure as the one used for the neutral scalar sector, we change to a new basis containing the Goldstone boson:

$$\begin{aligned}
G^+ &= \frac{1}{\beta} [-v_\eta e^{i\theta_\eta} \eta_1^+ + v_\rho \rho^+ + v_s e^{-i\theta_s} h_1^+] \\
V_1^+ &= \frac{1}{\alpha} [v_s e^{i\theta_\eta} \eta_1^+ + v_\eta e^{-i\theta_s} h_1^+] \\
V_2^+ &= \frac{1}{\alpha\beta} [v_\eta v_\rho e^{i\theta_\eta} \eta_1^+ + \alpha^2 \rho^+ - v_s v_\rho e^{-i\theta_s} h_1^+]
\end{aligned} \tag{15}$$

where the complex 3×3 mass matrix takes the form

$$M^2 = \frac{1}{2} \begin{pmatrix} 0 & 0 & 0 \\ 0 & A & B \\ 0 & B^* & C \end{pmatrix} \quad (16)$$

Up to the order (v_η^2/v_χ^2) , we find for the elements A , B and C in (16) the values

$$\begin{aligned} A &= f_1 \mu_1 v_\rho v_\chi \left(\frac{v_\eta \cos \theta_\eta}{\alpha^2} - \frac{1}{v_\eta \cos \theta_\eta} \right) \\ &\quad + f_2 \mu_2 v_\rho v_\chi \left(\frac{v_s \cos \theta_s}{\alpha^2} - \frac{1}{v_s \cos \theta_s} \right) - f_4 v_\chi^2 \left(\frac{v_s \cos \theta_s}{v_\eta \cos \theta_\eta} + \frac{v_\eta \cos \theta_\eta}{v_s \cos \theta_s} \right) \\ B &= \frac{\beta v_\chi}{\alpha^2} (-f_1 \mu_1 v_s e^{-i\theta_\eta} + f_2 \mu_2 v_\eta e^{i\theta_s}) \quad C = \frac{|B|^2}{A} \end{aligned} \quad (17)$$

Here, the CP violation is produced by the imaginary part of B . The relevant 2×2 complex submatrix in (16) can be easily diagonalized by means of

$$\mathcal{V}^{h^+} = \begin{pmatrix} \cos \gamma & -\sin \gamma e^{i\varphi} \\ \sin \gamma e^{-i\varphi} & \cos \gamma \end{pmatrix} \quad (18)$$

where $\tan 2\gamma = 2|B|/(C - A)$ and $\tan \varphi = \text{Im}B/\text{Re}B$. Thus, to this order, the physical charged scalars are given by

$$\begin{pmatrix} h^+ \\ H^+ \end{pmatrix} = \mathcal{V}^{h^+} \begin{pmatrix} V_1^+ \\ V_2^+ \end{pmatrix} \quad (19)$$

The squared mass of H^+ is order λv_χ^2 , while the other eigenvalue is zero. Going to the next order in v_η^2/v_χ^2 , the zero is not maintained, so we have $m_{h^+}^2 \sim \lambda v_\eta^2$. Notice that the parameter φ in \mathcal{V}^{h^+} is a CP violation source that will appear in the Yukawa couplings.

$SU(3)_L \otimes U(1)_Y$ effects on Δm_K , ε and ε'

The model under consideration was shown to present flavor changing neutral currents in the scalar sector, as well as Higgs-mediated interactions with CP violation. Hence, it is well possible to find nonnegligible contributions to the mass difference $m_{K_L} - m_{K_S}$ and the CP-violating parameters ε and ε' .

First of all, we consider the tree level contribution to Δm_K shown in Fig. 1, which has no analogous within the Standard Model. In the $SU(3)_L \otimes U(1)_Y$, the diagram arose since the couplings between the quarks and the physical scalars h_i^0 are nondiagonal in flavor. After the evaluation of the effective $\Delta S = 2$ Hamiltonian, and using $\Delta m_K = 2 \text{Re}\langle K^0 | \mathcal{H}_{eff} | \bar{K}^0 \rangle$ we obtain the value

$$(\Delta m_K)_{h^0} \simeq (\Delta_{12}^d)^2 \frac{4f_K^2 m_K^3}{3\alpha^2 N_2^2} B_K \sum_{i=1}^3 \frac{1}{m_{h_i^0}^2} [K_1 K_2 \mathcal{U}_{2i} \mathcal{U}_{3i} + \frac{1}{4}(K_1^2 - K_2^2)(\mathcal{U}_{2i}^2 - \mathcal{U}_{3i}^2) - \frac{m_d}{2m_s}(K_1^2 + K_2^2)(\mathcal{U}_{2i}^2 + \mathcal{U}_{3i}^2)] \quad (20)$$

Here, and in the following, we use the definitions

$$\begin{aligned} K_1 &= \frac{\beta}{v_\rho} c - \frac{v_s}{v_\eta} b \\ K_2 &= \frac{v_s}{v_\eta} d \\ K_3 &= \frac{\alpha^2}{\beta v_\rho} c \end{aligned}$$

with a , b , c and d as in (13). As is usually done, we used the vacuum insertion approximation to estimate the hadronic matrix element. The uncertainty is absorbed in the parameter B_K . It will be shown below that the value in (20) represents a severe constraint for the $SU(3)_L \otimes U(1)_Y$ model Higgs sector in connection with the flavor-changing effects.

Let us now analyze the model predictions for the ε and ε' parameters. According to standard definitions, we have

$$\varepsilon = \frac{\text{Im} M_{12}}{\sqrt{2} \Delta m_K} + \frac{1}{\sqrt{2}} \frac{\text{Im} A_0}{\text{Re} A_0} \quad (21a)$$

$$\varepsilon' = i \frac{e^{i(\delta_2 - \delta_0)}}{\sqrt{2}} \left(\frac{\text{Im} A_2}{\text{Re} A_0} - \frac{\text{Im} A_0 \text{Re} A_2}{(\text{Re} A_0)^2} \right) \quad (21b)$$

where $M_{12} = \langle K^0 | \mathcal{H}_{eff} | \bar{K}^0 \rangle$ and A_I is the K^0 decay amplitude to the final $\pi\pi$ state with isospin I . The δ_I stand for the corresponding phase shifts.

The lowest order neutral Higgs contribution to ε is obtained by taking the imaginary part of the diagram in Fig. 1. We get

$$(\text{Im}M_{12})_{h^0} = (\Delta_{12}^d)^2 \frac{f_K^2 m_K^3 B_K}{3\alpha^2 N_2^2} \sum_{i=1}^3 \frac{1}{m_{h_i^0}^2} \left[K_1 K_2 (\mathcal{U}_{2i}^2 - \mathcal{U}_{3i}^2) - (K_1^2 - K_2^2) \mathcal{U}_{2i} \mathcal{U}_{3i} \right] \quad (22)$$

The amplitude M_{12} also receives contributions from the charged Higgs. In the Appendix, we evaluate the relevant $H - W$ box diagram in Fig. 2, showing that it can be neglected when compared with the values in (20) and (22).

The neutral Higgs contribution to $\text{Im}A_0$ can be calculated from the tree level diagram in Fig. 3(a). As we just want to get an approximate value, it is reasonable to assume $(\text{Im}A_0)_{h^0} \simeq (\text{Im}A_2)_{h^0}$. This means that the $\pi\pi$ isospin and charge eigenstates give contributions of the same order. We use the $\pi^+\pi^-$ state, obtaining

$$\begin{aligned} (\text{Im}A)_{h^0} \simeq & -\Delta_{12}^d \frac{f_K m_\pi^2 m_K^2}{(m_u + m_d) \alpha^2 N_2^2} \sum_{i=1}^3 \frac{1}{m_{h_i^0}^2} \left\{ m_d [(1 - \Delta_{11}^d) K_1 K_2 (\mathcal{U}_{2i}^2 - \mathcal{U}_{3i}^2) \right. \\ & - K_2 K_3 \mathcal{U}_{2i}^2 + (K_2^2 (1 - \Delta_{11}^d) + K_1 (K_3 - K_1)) \mathcal{U}_{2i} \mathcal{U}_{3i}] \\ & \left. + \left[\begin{array}{c} m_d \leftrightarrow m_u \\ (1 - \Delta_{11}^d) \leftrightarrow \Delta_{11}^u \end{array} \right] - m_d K_1 \Delta_{11}^d \mathcal{U}_{2i} \mathcal{U}_{3i} \right\} \end{aligned} \quad (23)$$

We also need to consider the h^+ -mediated tree level diagrams shown in Fig. 3(b). Repeating the above approximation we obtain

$$\begin{aligned} (\text{Im}A)_{h^+} = & \frac{1}{8} \frac{f_\pi m_\pi^2 m_K^2}{(m_u + m_d) v_\eta v_\rho} \left(-\frac{v_s}{2\beta} \sin(2\gamma) \sin \varphi \right) \frac{1}{m_{h^+}^2} \\ & \times [m_d (\sin \theta_c (V_L^d)_{11} (V_L^u)_{11}) - \cos \theta_c (V_L^d)_{12} (V_L^u)_{11}) \\ & + m_u (\sin \theta_c (V_L^d)_{11} (V_L^u)_{11} + \cos \theta_c (V_L^d)_{12} (V_L^u)_{11} - \sin \theta_c \cos \theta_c)] \end{aligned} \quad (24)$$

Finally, to the same order of magnitude, it is necessary to evaluate the “penguin-like” diagram in Fig. 4. This conduces to

$$\begin{aligned} (\text{Im}A_0)_{peng} = & -\frac{1}{16\pi} \frac{m_c^2 m_s}{m_{h^+}^2} \frac{1}{v_\eta v_\rho} \left(\ln \left(\frac{m_{h^+}^2}{m_c^2} \right) - \frac{3}{2} \right) \left(\frac{v_s}{2\beta} \sin(2\gamma) \sin \varphi \right) \\ & \times (\sin \theta_c \cos \theta_c + \cos \theta_c (V_L^d)_{11} (V_L^u)_{12} - \sin \theta_c (V_L^d)_{12} (V_L^u)_{12}) \mathcal{M}_{peng} \end{aligned} \quad (25)$$

where

$$\mathcal{M}_{peng} = \frac{g_s}{4\pi} \langle \pi\pi | i \bar{s} \sigma^{\mu\nu} (1 - \gamma_5) \lambda^a d g_s \frac{q_\nu}{q^2} \bar{q}' \lambda^a \gamma_\mu q' | K^0 \rangle \quad (26)$$

(Notice that the penguin diagram contributes only to the $\Delta I = \frac{1}{2}$ amplitude.)

Numerical analysis and conclusions

We have estimated the contributions to Δm_K , ε and ε' coming from Higgs-mediated diagrams. Now, it is necessary to check the compatibility with the experimental data.

The value of Δm_K is accurately measured to be [7]

$$\Delta m_K = (3.510 \pm 0.018) \times 10^{-15} \text{GeV} \quad (27)$$

As this result agrees well with the Standard Model prediction, the Higgs presence should not introduce significant modifications. In order to see what happens, we refer to Eq. (20), where we take $f_K = 0.16 \text{ GeV}$, $m_K = 0.5 \text{ GeV}$ and $\alpha^2 \simeq 2 \times 10^4 \text{ GeV}^2$ (see relation (4)). After doing this, the number of unknown parameters is still very large. Nevertheless, in order to get an order of magnitude for $(\Delta m_K)_{h^0}$, it is reasonable to assume that the ratio K_i/N_2 and the mixing angles in \mathcal{U} are approximate to 1. We have then

$$(\Delta m_K)_{h^0} \sim (\Delta_{12}^d)^2 \frac{B_K}{m_{h_i^0}^2} 2 \times 10^{-7} \text{GeV}^3 \quad (28)$$

Using lattice calculations, the value of B_K is found to be between 1 and $\frac{1}{3}$, while the neutral Higgs masses are uncertain. We will take into account the theoretical bounds obtained for two-doublet models [8], hence we demand the value of m_{h_i} not to be greater than $\sim 150 \text{ GeV}$. Now, if $(\Delta m_K)_{h^0}$ is imposed to be lower than $(\Delta m_K)_{exp}$ at least by one order of magnitude we get the constraint

$$(\Delta_{12}^d) = (V_L^d)_{11}(V_L^d)_{12} \lesssim 0.01 \quad (29)$$

Notice that this constraint on V_L^d has nothing to do with CP violation, except for the fact that we have taken $\mu \approx v_\chi$.

It is important to mention that there is another contribution to Δm_K similar to that of Fig. 1. This is due to the presence of the neutral gauge boson Z'^0 , whose couplings with the ordinary quarks are also not diagonal in flavor. As in the neutral scalar sector, the

experimental value of Δm_K will also constrain the values of the mixing angles in V_L^d for a given value of $m_{Z'}$ [9].

The next estimation to consider is that of ε . This parameter is also accurately measured with the result

$$|\varepsilon| = (2.258 \pm 0.018) \times 10^{-3} \quad (30)$$

Let us consider the first term in (21a). Using (20) and (22), we have

$$\frac{(\text{Im} M_{12})_{h^0}}{\sqrt{2}\Delta m_K} = \frac{(\Delta m_K)_{h^0}}{4\sqrt{2}(\Delta m_K)} \mathcal{F} \quad (31)$$

where \mathcal{F} represents the ratio between both sums in the expressions of $(\text{Im} M_{12})_{h^0}$ and $(\Delta m_K)_{h^0}$.

The upper bound in (29) conduces to $\mathcal{F} \sim 0.1$. This does not seem to be reasonable, since both sums in (20) and (22) are in principle of the same order of magnitude^a. On the contrary, if we assume $\mathcal{F} \sim \mathcal{O}(1)$ we need for the angles in V_L^d the stronger constraint

$$(V_L^d)_{11}(V_L^d)_{12} \lesssim 4 \times 10^{-3} \quad (32)$$

The second term in (21a) can be treated using analogous approximations as in the derivation of (28). From Eqs. (23) and (24), our calculations lead to

$$(\text{Im} A)_{h^0} \sim -\Delta_{12}^d \times 10^{-12} \text{GeV} \quad (33a)$$

$$(\text{Im} A)_{h^+} \sim \mathcal{O}(10^{-14}) \text{GeV} \quad (33b)$$

These values have been obtained taking $m_{h^+} \sim m_{h^0} \sim 150$ GeV together with angles γ and φ of order unity. As is discussed above, there is no reason to think that the tree level

^aThis is not the case in the limit when the scalars h_i^0 are degenerate in mass: due to the unitarity of \mathcal{U} , it is immediate from (22) that the contribution to M_{12} vanishes. Going to the next order in $(v_\eta/v_\chi)^2$, we could find a value for ε which is compatible with (29).

diagrams conducing to (33) produce a significant enhancement in one of the amplitudes A_I . Hence, we take in both cases $\text{Im}A_0 \sim \text{Im}A_2 \sim \text{Im}A$.

The penguin diagram in Fig. 4 contributes solely to $\text{Im}A_0$. From (25), the numerical estimation gives

$$\text{Im}A_0 \sim \mathcal{O}(10^{-11} - 10^{-12})\text{GeV} \quad (34)$$

where the uncertainty is fundamentally due to the mixing angles combinations. The value of \mathcal{M}_{peng} has been taken from bag model calculations [10],

$$\mathcal{M}_{peng} = 2.0\alpha_s\text{GeV}^2$$

with $\alpha_s \sim 0.2$.

Now, taking into account the experimental value $\text{Re}A_0 \simeq 3.3 \times 10^{-7} \text{ GeV}$, one can immediately see that both the tree level and penguin contributions to ε become vanishingly small. Thus, if we demand the complex vacuum expectation values to be the sole CP violation source in the theory, the experimentally measured value of ε has to be given by the contribution from $(\text{Im}M_{12})_{h_i^0}$. That is, the mixing angles in V_L^d must reach the upper limit in (32). The constraint is less severe in the limit when the three “light” neutral scalars have approximate masses.

Let us finally examine the ε' parameter. Here, the experimental data are not conclusive: different measurements give for ε'/ε [7]

$$\text{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) = \begin{cases} (2.3 \pm 0.65) \times 10^{-3} & \text{NA31} \\ (0.74 \pm 0.52 \pm 0.29) \times 10^{-3} & \text{E731} \end{cases} \quad (35)$$

the latter being still compatible with the “superweak” mixing, where $\varepsilon' = 0$. However, it is clear that both results in (35) imply an upper bound for $|\varepsilon'/\varepsilon|$ of order 10^{-3} .

To see what happens in the $SU(3)_L \otimes U(1)_Y$ model, we refer to Eq. (21b). Here, taking the above experimental values for $\text{Re}A_0$ and $|\varepsilon|$, the ratio $\text{Re}A_2/\text{Re}A_0 \simeq 1/22$ and the estimates in (33) and (34), we have

$$\left| \frac{\varepsilon'}{\varepsilon} \right|_{SU(3)} \sim \mathcal{O}(10^{-4}) \quad (36)$$

Thus, once we have fixed the value of ε , both Δm_K and $|\varepsilon'|$ fall very well within the experimental bounds.

It is interesting to compare our results with those obtained in Ref. [11] for a two-Higgs-doublet model. There, the authors introduced arbitrary small parameters in the Lagrangian in order to reproduce the observed CP violation and flavor changing phenomenological effects. The required values for the parameters were tabulated in terms of the Higgs masses and the VEV ratio v_1/v_2 . In the $SU(3)_L \otimes U(1)_Y$ model, the corresponding “small” parameter appears naturally as a mixing angle in the matrix V_L^d , as shown in Eq. (32). Notice that although both the $SU(3)_L \otimes U(1)_Y$ and the two-doublet models are remarkably different, the result in Eq. (32) is comparable with the bounds in Ref. [11].

The reason for the small value of $V_{11}^d V_{12}^d$ remains hidden. However, the observed hierarchies among the mixing angles in $V_{CKM} = V_L^{u\dagger} V_L^d$ suggest the presence of new underlying physics relating quark families, masses and mixing angles. If the $SU(3)_L \otimes U(1)_Y$ were the theory beyond the Standard Model, the mechanism responsible for a small angle compatible with (32) could probably come to light within this context.

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APPENDIX:

We estimate here the contribution to Δm_K and ε coming from the box diagram shown in Fig. 2. The relevant quark-scalar vertices in this graph are given by the interaction Lagrangian

$$\begin{aligned} \mathcal{L} = \overline{D} \left[\left(\frac{C_{22}^h}{v_\rho} \times \mathbb{1}^{(3 \times 3)} + \left(\frac{C_{12}^h}{v_\eta} - \frac{C_{22}^h}{v_\rho} \right) \Delta^d \right) V_{CKM}^\dagger \hat{M}^u \frac{(1 + \gamma_5)}{2} \right. \\ \left. + \hat{M}^d V_{CKM}^\dagger \left(\frac{C_{12}^h}{v_\eta} \times \mathbb{1}^{(3 \times 3)} - \left(\frac{C_{12}^h}{v_\eta} - \frac{C_{22}^h}{v_\rho} \right) \Delta^u \right) \frac{(1 - \gamma_5)}{2} \right] U h^- + \text{h.c.} \end{aligned} \quad (\text{A1})$$

where C^h represents the unitary matrix changing from the $(\eta_1^+, \rho^+, h_1^+)$ basis to the mass eigenbasis (G^+, h^+, H^+) . In terms of the angles γ and ϕ defined in (18), we have

$$C_{12}^h = \frac{e^{i\theta_\eta}}{\alpha} \left(v_s \cos \gamma - \frac{v_\eta v_\rho}{\beta} \sin \gamma e^{-i\varphi} \right) \quad C_{22}^h = -\frac{\alpha}{\beta} \sin \gamma e^{-i\varphi} \quad (\text{A2})$$

The calculation of this type of diagrams has been performed in Refs. [12] and [11], concerning the study of the Weinberg–Branco model and the two-Higgs models respectively. We will not repeat the details here. However, notice that in the $SU(3)_L \otimes U(1)_Y$ case there is no discrete symmetry (exact or approximate) preventing the presence of FCNI's. As a consequence, a contribution to the effective Lagrangian which does not appear in Refs. [12] and [11] is allowed. This new term is proportional to the Lorentz scalar operator

$$\bar{s}_L(q_1) \gamma^\alpha d_L(q_2) \bar{s}_L(p_1) \gamma_\alpha d_L(p_2) \quad (\text{A3})$$

Following the same steps as in Ref. [12] for the calculation of the $K^0 - \bar{K}^0$ amplitudes, we find

$$\begin{aligned} (\text{Im} M_{12})_{box} = \frac{G_F}{\sqrt{2}} \frac{1}{12\pi^2} \sin \theta_c \cos \theta_c \frac{m_K f_K^2 m_c^2}{m_{h^+}^2} \frac{\text{Im}(C_{12}^h C_{22}^{h*})}{v_\eta v_\rho} \\ \times \left[\left(\ln \left(\frac{m_c^2}{m_W^2} \right) + \frac{m_W^2}{m_h^2 - m_W^2} \ln \left(\frac{m_W^2}{m_h^2} \right) \right) m_c^2 (\sin \theta_c (V_L^d)_{12} (V_L^u)_{12} + \cos \theta_c (V_L^d)_{11} (V_L^u)_{12}) \right. \\ \left. + \frac{1}{8} m_K^2 \ln \left(\frac{m_c^2}{m_W^2} \right) \left(1 + 3 \frac{m_s^2}{m_K^2} \right) (\sin \theta_c (V_L^d)_{12} (V_L^u)_{12} - \cos \theta_c (V_L^d)_{11} (V_L^u)_{12} - \sin \theta_c \cos \theta_c) \right] \end{aligned} \quad (\text{A4})$$

Except for very small values of the angles in $V_L^{d,u}$, we see that the value of $(\text{Im} M_{12})_{box}$ is dominated by the first term in (A4). This is precisely the contribution due to the operator

(A3). Now considering the constraint given by (32) and all masses as in the evaluation of $(M_{12})_{h^0}$, we find that the value of $(\text{Im}M_{12})_{box}$ can be at most one order of magnitude smaller than the contribution of (22). On the other hand, the real part of $(M_{12})_{box}$ cannot be significantly different from the imaginary part, thus the box contribution to Δm_K may be safely neglected.

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FIGURES

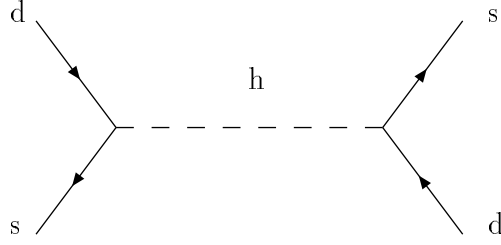


FIG. 1. Tree level contribution to Δm_K and ε due to neutral Higgs boson exchanges.

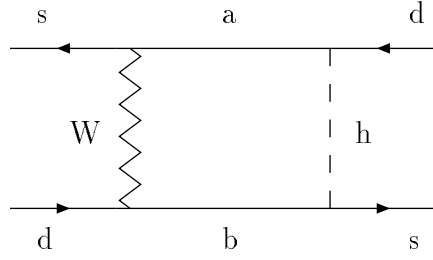


FIG. 2. Box diagram with one charged vector boson and one charged scalar boson exchange.

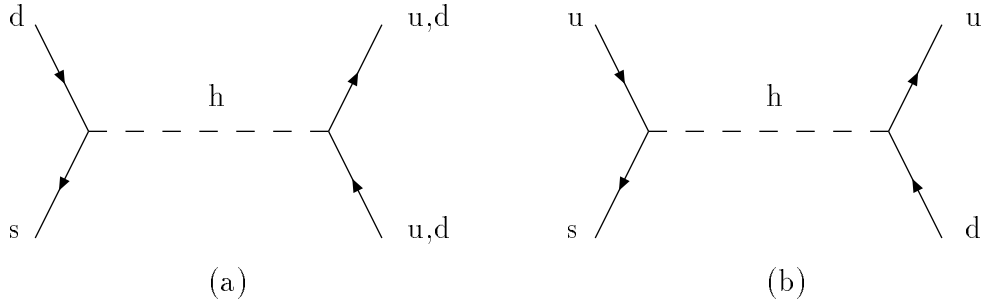


FIG. 3. (a) Tree level contribution to ε' due to the exchange of a neutral Higgs boson.

(b) Tree level contribution to ε' due to the exchange of a charged Higgs boson.

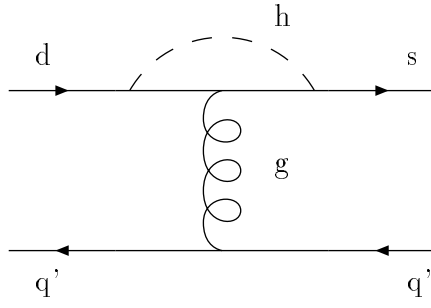


FIG. 4. Gluon penguin diagram mediated by a charged Higgs boson.